Automatic Control (1)

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Course Title: Automatic Control (1)

Course Code: EEC 321

Prerequisites: Signals and Systems

Study Hours: 3 Cr. hrs.

= [2 Lect. + 2 Tut + 0 Lab]





Final Exam: 40%.

Midterm: 1st 20% & 2nd 20%.

Year Work & Quizzes: 15%.

MATLAB Mini Project: 5%.

<u>Textbook:</u>

- 1. Nise, N. S. "Control System Engineering", 7th edition, John Wiley & Sons Ltd., UK, 2016.
- 2. Katsuhiko Ogata, "Modern Control Engineering", Prentice Hall, 5th Edition.
- 3. F. Golnaraghi and B. C. Kuo, "Automatic control Systems", 10th ed., John Wiley & Sons, Inc. 2017.
- 4. Andrea Bacciotti, "Stability and Control of Linear Systems" Volume 185, Springer, 2019



Course Description

- Introduction to control systems: Open loop systems, closed loop systems, transfer function and concept of poles and zeros. Block diagram reduction techniques. Signal flow graph techniques. Modeling of some electrical, mechanical and thermal systems. Time response of first and second order systems, PID controllers. Steady-state error. Concept of stability: Routh's stability criterion.
- Root locus method. State- space representation: State model of linear systems using physical variables, State, space representation using phase variables. Properties of transition matrix and solution of state equation. compensation. Programable

Examples

Sketch the root locus for the system $G(s)H(s) = \frac{K}{(s-1)(s+2)(s+3)}$ with the characteristic equation of;

 p_2

-1

 p_1

 p_3

- 1. Number of poles = n = 3.
- 2. Number of zeros = m = 0.
- 3. Number of asymptotes = n m = 3.
- 4. The RL start from pole s = 1, s = -2, and s = -3.

The RL occurs on the real axis for − 2< s <1, and s<−3.
 Because the number of roots is odd.

6. The real-axis asymptotes intercept is evaluated as;

$$\sigma_{a} = \frac{\sum poles - \sum zeros}{n - m}$$

$$\sigma_{a} = \frac{(1) + (-2) + (-3) - 0}{3 - 0} = \frac{-4}{3} = -1.33$$

• The angles of the asymptotes that intersect at - 3, given by :

$$\theta_{a} = \frac{(2i+1)\pi}{n-m} = \frac{(2i+1)\pi}{3-0}$$
For $i=0$, $\theta_{a} = 60^{\circ}$
For $i=1$, $\theta_{a} = 180^{\circ}$
For $i=2$, $\theta_{a} = 300^{\circ}$
For $i=2$

 $| \rangle$

7. The breakaway point, σ , can be determine as;



8. Find the points where the RL may cross the imaginary-axis.

The characteristic equation:

$$1+G(s)H(s) = 1 + \frac{K}{(s-1)(s+2)(s+3)} = 0$$

(s-1)(s+2)(s+3) + K = 0
s³ + 5s² + 6s - s² - 5s - 6 + K = 0
s³ + 4s² + s + K - 6 = 0
s = j\omega
(j\omega)³ + 4(j\omega)² + (j\omega) + K - 6 = 0
- j\omega³ - 4\omega² + j\omega + K - 6 = 0
- j\omega³ = 0
\omega³ = \omega = +1

 $\omega = \pm 1$



Control Systems Design Using Root-locus Approach

Control Design Using Root Locus

- <u>Case study</u>: double integrator, transfer function $G_s = \frac{1}{s^2}$
- <u>Control objective</u>: ensure stability, meet time response specifications.
- First, let's try a simple P-gain:



• Closed-loop transfer function:

$$\frac{\frac{K}{s^2}}{1+\frac{K}{s^2}} = \frac{K}{s^2+K}$$

Double Integrator with P-Gain $R \xrightarrow{+} \bigcirc \longrightarrow K \xrightarrow{1} \xrightarrow{1} \xrightarrow{1} Y$

Closed-loop transfer function:



- Characteristic equation: $s^2 + K = 0$
- Closed-loop poles: $s = \pm j \int k$

This confirms what we already knew: P-gain alone does not deliver stability.

-Re

Im

0

double pole

Double Integrator with PD-Control

$$R \xrightarrow{+} \bigcirc K_{\rm P} + K_{\rm D}s \xrightarrow{} \frac{1}{s^2} \xrightarrow{} Y$$

Characteristic equation:

$$1 + (K_P + K_D s) \cdot \frac{1}{s^2} = 0$$

 $s^2 + K_D s + K_P = 0$

• To use the Root-Locus method, we need to convert it into the Evans form 1 + KL(s) = 0, where $L(s) = \frac{b(s)}{a(s)}$

$$1 + (K_P + K_D s) \cdot \frac{1}{s^2} = 1 + K_D \frac{s + \frac{K_P}{K_D}}{s^2} = 0$$

$$\Rightarrow K = K_D, \qquad L(s) = \frac{s + \frac{K_P}{K_D}}{s^2} \qquad (assume \ \frac{K_P}{K_D} \ fixed, = 1)$$

But let's actually draw the RootLocus using the rules:



- What can we conclude from this root locus about stabilization?
- all closed-loop poles are in LHP (we already knew this from Routh, but now can visualize).
- > nice damping, so can meet reasonable specifications.

So, the effect of D-gain was to introduce an open-loop zero into LHP, and this zero "pulled" the root locus into LHP, thus stabilizing the system.

Dynamic Compensation

 <u>Objectives</u>: stabilize the system and satisfy given time response specifications using a stable, causal controller.



Characteristic equation:

$$1 + K \frac{s+z}{s+p} \cdot \frac{1}{s^2} = 1 + K L(s) = 0$$

Approximate PD Using Dynamic Compensation

 <u>Reminder</u>: we can approximate the D-controller K_D s by:

$$\frac{K_D}{s+p} \implies K_D s \quad as \ p \to \infty$$

where, p is the pole of the controller.

• So, we replace the PD controller $K_P + K_D s$ by:



• Closed-loop poles: $1 + (Kp + KD \frac{PS}{S+p})$

Lead & Lag Compensators Consider a general controller of the form

 $K \frac{ps}{s+p} \implies K, z, p > 0$ are design parameters

Depending on the relative values of z and p, we call it: a lead compensator when z < p</p> > a lag compensator when z > p

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- Why the name "lead/lag?"
- Think frequency response

$$\angle \frac{j\omega + z}{j\omega + p} = \angle (j\omega + z) - \angle (j\omega + p) = \psi - \phi$$

* if z < p, then $\psi - \varphi > 0$ (phase lead). * if z > p, then $\psi - \varphi < 0$ (phase lag).

p

z



• Controller transfer function is $K_{\frac{s+h}{s+h}}^{\frac{s}{s+h}}$, where

$$K = K_{\rm P} + pK_{\rm D}, \qquad z = \frac{pK_{\rm P}}{K_{\rm P} + pK_{\rm D}} \xrightarrow{p \to \infty} \frac{K_{\rm P}}{K_{\rm D}}$$

• so, as $p \to \infty$, z tends to a constant, so we get a <u>lead</u> <u>controller</u>.

We use lead controllers as dynamic compensators for approximate PD control

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Double Integrator & Lead Compensator



- To keep things simple, let's set K_p= K_D. Then:
 K = K_p+pK_D = (1+p)K_D
 z = pK_p/K_p = pK_D/(1+p)K_D = p/(1+p) ≈ 1 as p → ∞
 Since we can choose p and z directly, let's take
 z = 1 and p = large.
 - We expect to get behavior similar to PD control.

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$$L(s) = \frac{s+z}{s+p} \cdot \frac{1}{s^2} \stackrel{z=1}{=} \frac{s+1}{s^2(s+p)}$$

Let's try a few values of p. Here's p = 10:



 Close to jw-axis, this root locus looks similar to the PD root locus. However, the pole at s = - 10 makes the locus look different for s far into LHP.

- The design seems to look good: nice damping, can meet reasonable specs.
- Any concerns with large values of p?
- When p is large, we are very close to PD control, so we run into the same issue: noise amplification.
- (This is just intuition for now we will confirm it later using frequency-domain methods.)

Control Design Using Root Locus

<u>Case study</u>: double integrator, transfer function.

$$Gp(s) = \frac{1}{s-1}$$

- <u>Control objective</u>: ensure stability and constant reference tracking.
- In earlier lectures, we saw that for perfect steady-state tracking we need PI control :



Closed-loop poles are determined by the Characteristic equation:

$$1 + \left(K_P + \frac{K_I}{s}\right) \left(\frac{1}{s-1}\right) = 0$$

• To use the Root-Locus method, we need to convert it into the Evans form 1 + KL(s) = 0, where $L(s) = \frac{b(s)}{a(s)}$

$$1 + \left(K_{P} + \frac{K_{I}}{s}\right)\left(\frac{1}{s-1}\right) = 1 + \frac{K_{P}s + K_{I}}{s}\left(\frac{1}{s-1}\right) = 1 + K_{P}\frac{s + \frac{K_{I}}{K_{P}}}{s(s-1)} = 0$$

$$\Rightarrow K = K_{P}, \qquad L(s) = \frac{s + \frac{K_{I}}{K_{P}}}{s(s-1)} \qquad (assume\frac{K_{I}}{K_{P}}fixed, = 1)$$

Root Locus for PI Compensation

- The system is stable for K > 1 (from Routh-Hurwitz).
- For very large K, we get a completely damped system, with negative real poles.
- Perfect steady-state tracking references.



Approximate PI via Dynamic Compensation

- PI control achieves the <u>objective of stabilization</u> and <u>perfect steady state</u> tracking of constant references; however, just as with PD earlier, we want a stable controller.
- Here's an idea:

replace
$$K \frac{s+1}{s}$$
 by $K \frac{s+1}{s+p}$, where p is small

• More generally, if $z = \frac{K1}{K_p}$, then

replace
$$K \frac{s+z}{s}$$
 by $K \frac{s+z}{s+p}$, where $p < z$

This is lag compensation (or lag control).
 We use lag controllers as dynamic compensators for approximate P25 control.



Design by Root-Locus Method

- The root-locus approach to design is very powerful when the specifications are given in terms of timedomain quantities, such as the damping ratio and undamped natural frequency of the desired dominant closed-loop poles, maximum overshoot, rise time, and settling time.
- The design by the root-locus method is based on reshaping the root locus of the system by adding poles and zeros to the system's open-loop transfer function and forcing the root loci to pass through desired closed-loop poles in the s plane.

- Consider a design problem in which the original system either is unstable for all values of gain or is stable but has undesirable transient response characteristics.
- In such a case, the reshaping of the root locus is necessary in the broad neighborhood of the $j\omega$ axis and the origin in order that the dominant closed-loop poles be at desired locations in the complex plane. This problem may be solved by inserting an appropriate lead compensator in cascade with the feed-forward transfer function.

Series Compensation & Parallel Compensation

a) Series Compensation:



a) Parallel or feedback Compensation



Commonly Used Compensators

Lead Compensator

If a sinusoidal input is applied to the input of a network, and the steady state output (which is also sinusoidal) has a phase lead.

Lag Compensator

If a sinusoidal input is applied to the input of a network, and the steady state output (which is also sinusoidal) has a phase lag.

Lead-Lag Compensator

If a sinusoidal input is applied to the input of a network, and the steady state output (which is also sinusoidal) has both phase lag and phase lead occur in the output but in different frequency regions; phase lag occurs in the low-frequency region and phase lead occurs in the high-frequency region.

Effects of the Addition of Poles

The addition of a pole to the open-loop transfer function has the effect <u>of pulling the root locus to</u> <u>the right</u>, tending to lower the system's relative stability and to <u>slow down the settling of the</u> <u>jw</u>

(a) Root-locus plot of
a single-pole system.
(b) root-locus plot
of a two-pole system.
(c) root-locus plot
of a three-pole system.



Effects of the Addition of Zeros

The addition of a zero to the open-loop transfer function has the effect of pulling the root locus to the left, tending to <u>make the system more stable</u> and <u>to speed up the settling of the response</u>. (Physically, the addition of a zero in the feed-forward transfer function means the addition of derivative control to the system. The effect of such control is to <u>introduce a degree of anticipation into the system</u> <u>and speed up the transient response.</u>)



Lead or Lag Compensation

The transfer function is



Where:

$$T = R_1 C_1, \qquad \alpha T = R_2 C_2, \qquad K_c = \frac{R_4 C_1}{R_3 C_2} \qquad K_c \alpha = \frac{R_4 C_1}{R_3 C_2} \frac{R_2 C_2}{R_1 C_1} = \frac{R_2 R_4}{R_1 R_3}, \qquad \alpha = \frac{R_2 C_2}{R_1 C_1}$$

This network has a dc gain of

$$K_c \alpha = \frac{R_2 R_4}{R_1 R_3},$$

• <u>This network is</u> • a lead network if $R_1C_1 > R_2C_2$

> a lag network if

$$R_1C_1 < R_2C_2$$

Lead Compensation Techniques Based on the Root-Locus Approach

- The designing a lead compensator for the system shown by the root-locus method may be stated as follows: $\xrightarrow{G_c(s)} \xrightarrow{G_c(s)} \xrightarrow{G(s)} \xrightarrow{G(s$
- 1. From the performance specifications, determine the desired location for the dominant closed-loop poles.
- 2. By drawing the root-locus plot of the uncompensated system (original system), ascertain whether or not the gain adjustment alone can yield the desired closed-loop poles. <u>If not, calculate the angle deficiency \u03c6</u>. This angle must be contributed by the lead compensator if the new root locus is to pass through the desired locations for the dominant closed-loop poles.

3. Assume the lead compensator Gc(s) to be

$$G_c(s) = K_c \alpha \frac{Ts+1}{\alpha Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}, \qquad (0 < \alpha < 1)$$

where α and T are determined from the angle deficiency. Kc is determined from the requirement of the open-loop gain.

4. If static error constants are not specified, determine the location of the pole and zero of the lead compensator so that the lead compensator will contribute the necessary angle φ . If no other requirements are imposed on the system, try to make the value of α as large as possible. A larger value of a generally results in a larger value of Kv, which is desirable. Note that

$$K_v = \lim_{s \to 0} sG_c(s)G(s) = K_c \alpha \lim_{s \to 0} sG_c(s)$$

5. Determine the value of Kc of the lead compensator from the magnitude condition.

EXAMPLE

 Consider the position control system with the feedforward transfer function is:



• It is desired to design a lead compensator Gc(s), so that the dominant closed-loop poles have the damping ratio $\eta = 0.5$ and the undamped natural frequency $\omega n = 3 rad/sec$.



Solution

The closed-loop transfer function for the system is:

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + s + 10} = \frac{10}{(s + 0.5 + j3.1225)(s + 0.5 - j3.1225)}$$

The closed-loop poles are located at:

$$s = -0.5 \pm j3.1225$$

The damping ratio of the closed-loop poles is η = 0.5.
 The undamped natural frequency of the closed-loop poles is ωn = 3 rad/sec.

Because the damping ratio is small, this system will have a large overshoot in the step response and is not desirable.



The desired location of the dominant closed-loop poles can be determined from:

$$s^{2} + 2\zeta \omega_{n}s + \omega_{n}^{2} = s^{2} + 3s + 9$$

= $(s + 1.5 + j2.5981)(s + 1.5 - j2.5981)$
as follows:
$$s = -1.5 \pm j2.5981$$

-1.5

-3

Solution (Method -1)

- First, determine the necessary angle φ to be added so that the total sum of the angles is equal to \pm 180(2k + 1).
- > The angle from the pole at the origin to the desired dominant closed-loop pole at s = -1.5 + j2.5981 is 120°. The angle from the pole at s = -1 to the desired closed-loop pole is 100.894°. Hence, the angle deficiency is:

Angle deficiency = $180^{\circ} - 120^{\circ} - 100.894^{\circ} = -40.894^{\circ}$

> Thus, if we need to force the root locus to go through the desired closed-loop pole, the lead compensator must contribute $\varphi = 40.894^{\circ}$ at this point.



Then we bisect angle APO and take 40.894°/2 each side, then the locations of the zero and pole are found as follows:

zero at
$$s = -1.9432$$

pole at $s = -4.6458$



Thus, Gc(s) can be given as:

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = K_c \frac{s + 1.9432}{s + 4.6458}$$

- For this compensator the value of a is a = 1.9432/4.6458 = 0.418.
- The value of *Kc* can be determined by use of the magnitude condition.

$$\left| K_c \frac{s+1.9432}{s+4.6458} \frac{10}{s(s+1)} \right|_{s=-1.5+j2.5981} = 1$$

$$K_c = \left| \frac{(s + 4.6458)s(s + 1)}{10(s + 1.9432)} \right|_{s = -1.5 + j2.5981} = 1.2287$$

 Hence, the lead compensator Gc(s) just designed is given by

$$G_c(s) = 1.2287 \frac{s+1.9432}{s+4.6458}$$
⁴⁵

the closed-loop transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{12.287s + 23.876}{s^3 + 5.646s^2 + 16.933s + 23.876}$$

Solution (Method -2)

• If we choose the zero of the lead compensator at s = -1 so that it will cancel the plant pole at s = -1, then the compensator pole must be located at s = -3.

$$G_c(s) = K_c \frac{s+1}{s+3}$$

 The value of Kc can be determined by use of the magnitude condition.

$$\left| K_c \frac{s+1}{s+3} \frac{10}{s(s+1)} \right|_{s=-1.5+j2.5981} = 1$$
$$K_c = \left| \frac{s(s+3)}{10} \right|_{s=-1.5+j2.5981} = 0.9$$

 Hence, the lead compensator Gc(s) just designed is given by



Lead Compensation Techniques Based on the Root-Locus Approach

 The designing a lag compensator for the system shown by the root locus
 method may be stated as follows:



- 1. Draw the root-locus plot for the uncompensated system whose open-loop transfer function is G(s). Based on the transient-response specifications, locate the dominant closed-loop poles on the root locus.
- 2. Assume the lag compensator Gc(s) to be

$$G_{c}(s) = \hat{K}_{c}\beta \frac{Ts+1}{\beta Ts+1} = \hat{K}_{c} \frac{s+\frac{1}{T}}{s+\frac{1}{s+1}} \qquad (\beta > 1)$$

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Then the open-loop transfer function of the compensated system becomes Gc(s), G(s).

- 3. Evaluate the particular static error constant specified in the problem.
- 4. Determine the amount of increase in the static error constant necessary to satisfy the specifications.
- 5. Determine the pole and zero of the lag compensator that produce the necessary increase in the particular static error constant without appreciably altering the original root loci. (Note that the ratio of the value of gain required in the specifications and the gain found in the uncompensated system is the required ratio between the distance of the zero from the origin and that of the pole from the origin.)
- 6. Adjust gain *Kc* of the compensator from the magnitude condition so that the dominant closed-loop poles lie at the desired location.

 $(\hat{K}_c \text{ will be approximately } 1.)$

Example

• Consider the control system shown. Design a lag compensator Gc(s) such that the static velocity error constant kv is 50 sec-1 without appreciably changing the location of the original closed-loop poles, which are at $s = -2 \pm j16$.



Solution

- Assume that the transfer function of the lag compensator is $G_c(s) = \hat{K}_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} \qquad (\beta > 1)$
- Since k_v is specified as 50 sec⁻¹, we have $K_v = \lim_{s \to 0} sG_c(s) \frac{10}{s(s+4)} = \hat{K}_c \beta 2.5 = 50$
- Thus $\hat{K}_c \beta = 20$
- Now choose $\hat{K}_c = 1$, then $\beta = 20$

• Choose T = 10. Then the lag compensator can be given by

$$G_c(s) = \frac{s + 0.1}{s + 0.005}$$

• The angle contribution of the lag compensator at the closed-loop pole $s = -2 \pm j16$.

$$\underline{/G_c(s)}\Big|_{s=-2+j\sqrt{6}} = \tan^{-1}\frac{\sqrt{6}}{-1.9} - \tan^{-1}\frac{\sqrt{6}}{-1.995} = -1.3616^\circ$$

 which is small. The magnitude of Gc(s) at is 0.981. Hence the change in the location of the dominant closed-loop poles is very small. The open-loop transfer function of the system becomes

$$G_c(s)G(s) = \frac{s+0.1}{s+0.005} \frac{10}{s(s+4)}$$

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{10s + 1}{s^3 + 4.005s^2 + 10.02s + 1}$$

A unity feedback control system has: $G(s) = \frac{K(s+1)}{s(s+20)}$ and r(t) = 1Find the value of K so that the

steady-state error does not exceed (0.01). With the calculated value of K, test the system stability.

With Our Best Wishes Automatic Control (1) Course Staff



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